

Paper Code : 2303

Ph. D. (Mathematics) Entrance Examination 2023-24

Time : 2 Hours

Do not open this Test Booklet until you are asked to do so.

Maximum Marks : 100

Immediately fill in the particulars on this page of the Question Booklet and the Answer Sheet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.

Name of the candidate (In Capital Letters)

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(In Words) :

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Important Instructions :

Invigilator's Signature _____

This booklet contains 100 objective type questions. First 50 questions of Research Methodology and Second 50 questions of Subject Specific. Each having four options a, b, c, d.

1. Candidates are not allowed to carry textual material printed or written, bits of papers, pages, mobile phone, electronic device or any other material except the Admit Card inside the Examination Hall/Room.
2. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet with Blue/Black Ball Point Pen.
3. The candidate should not write their Roll Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.
4. Out of the four options given for each question, the candidate must mark one correct option as an answer only.
5. There is no negative marking for any wrong answer.
6. Handle the Test Booklet and Answer Sheet with care, as under no circumstances (except for discrepancy in the Test Booklet Code and Answer Sheet Code), will another set be provided.
7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/writing work are to be done in the space available in the Test Booklet itself.
8. Each candidate must show on demand her Admit Card to the Invigilator.
9. No candidate, without special permission of the Superintendent or Invigilator, should leave her seat.
10. The candidate should not leave the Examination Hall without handing over their Answer Sheet and Test Booklet to the invigilator on the duty and signing the Attendance Sheet.
11. No part of the Test Booklet and Answer Sheet shall be detached under circumstances.

NOTE : THIS BOOKLET SHALL BE RETAINED FOR SIX MONTH ONLY.

RESEARCH METHODOLOGY

1. What is the name of the conceptual framework in which the research is carried out ?
 - (a) Research hypothesis
 - (b) Synopsis of research
 - (c) Research paradigm
 - (d) Research design
2. What is the major attribute of Correlation Analysis ?
 - (a) Association among variables
 - (b) Difference among variables
 - (c) Regression among variables
 - (d) Variations among variables
3. Which of the following features are considered as critical in qualitative research ?
 - (a) Collecting data with the help of standardized research tools.
 - (b) Design sampling with probability sample techniques.
 - (c) Collecting data with bottom-up empirical evidence.
 - (d) Gathering data with top-down schematic evidence.
4. How is random sampling helpful ?
 - (a) Reasonably accurate
 - (b) An economical method of data collection
 - (c) Free from personal biases
 - (d) All of the above

5. A research intends to explore the result of possible factors for the organization of effective mid-day meal interventions. Which research method will be most appropriate for this study ?
- (a) Descriptive survey method
 - (b) Historical method
 - (c) Ex-post facto method
 - (d) Experimental method
6. In order to pursue the research, which of the following is priorly required ?
- (a) Developing a research design
 - (b) Formulating a research question
 - (c) Deciding about the data analysis procedure
 - (d) Formulating a research hypothesis
7. The format of thesis writing is the same as in :
- (a) Writing of Seminar representation
 - (b) Preparation of research paper/article
 - (c) A research dissertation
 - (d) Presenting a workshop/conference paper
8. Which one among the following statements is false in the context of participatory research ?
- (a) It recognizes knowledge as power
 - (b) It is a collective process of inquiry
 - (c) It emphasizes people as experts
 - (d) Its sole purpose is the production of knowledge

9. Which one among the following statements is *true* in the context of the testing of hypotheses ?
- (a) It is only the alternative hypotheses that can be tested.
 - (b) It is only the null hypotheses that can be tested.
 - (c) Both the alternative and the null hypotheses can be tested.
 - (d) Both the alternative and the null hypotheses cannot be tested.
10. Research and Development become the index of development of the country. Which of the following reasons are true with regards to this statement?
- (a) R&D targets human development
 - (b) R&D can enhance people's standard of living in the country
 - (c) R&D reflects the actual economic and social conditions being prevailed in the country
 - (d) All of the above
11. What does the longitudinal research approach actually deal with ?
- (a) Long-term research
 - (b) Short-term research
 - (c) Horizontal research
 - (d) None of the above
12. What do you understand by the term "Anusandhan" ?
- (a) Goal-oriented
 - (b) Following an aim
 - (c) Attaining an aim
 - (d) Praying to achieve an aim

- 13.** Which of the following does not correspond to characteristics of research?
- (a) Research is not passive
 - (b) Research is systematic
 - (c) Research is not problem-oriented
 - (d) Research is not a process
- 14.** Which of the following options are the main tasks of research in modern society ?
- (a) To learn new things
 - (b) To keep pace with the advancement in knowledge
 - (c) To systematically examine and critically analyze the investigations/ sources with the objective
 - (d) All of the above
- 15.** What is the main aim of interdisciplinary research ?
- (a) To oversimplify the problem of research
 - (b) To bring out the holistic approach to research
 - (c) To create a new trend in research methodology
 - (d) To reduce the emphasis on a single subject in the research domain
- 16.** The main aim of the scientific method in the research field is to :
- (a) Improve data interpretation
 - (b) Confirm triangulation
 - (c) Eliminate spurious research
 - (d) Introduce new variables

17. A researcher is interested in studying the prospects of a particular political party in an urban area. So, what tool should he prefer for the study ?
- (a) Rating Scale (b) Questionnaire
(c) Interview (d) Schedule
18. The conclusions/findings of which type of research cannot be generalized to other situations ?
- (a) Casual Comparative Research
(b) Historical Research
(c) Descriptive Research
(d) Experimental Research
19. How to judge the depth of any research ?
- (a) By research title
(b) By research duration
(c) By research objectives
(d) By total expenditure on research
20. Who can successfully conduct Research ?
- (a) Someone who is a hard worker
(b) Someone who possesses post-graduation degree
(c) Someone who has studied research methodology
(d) Someone who possesses thinking and reasoning ability

21. Which of the following is *not* the method of Research ?

- (a) Survey
- (b) Historical
- (c) Observation
- (d) Philosophical

22. A research problem is feasible only when :

- (a) It has utility and relevance
- (b) It is new and adds something to knowledge
- (c) It is researchable
- (d) All of the above

23. Circle graphs are used to show :

- (a) How is one part related to other parts ?
- (b) How are various sections share in the whole ?
- (c) How is one whole related to another whole ?
- (d) How are various parts related to the whole ?

24. Authenticity of a research finding is its :

- (a) Validity
- (b) Objectivity
- (c) Originality
- (d) All of these

25. Which one is called non-probability sampling ?

- (a) Quota sampling
- (b) Cluster sampling
- (c) Systematic sampling
- (d) Stratified random sampling

26. What does a good thesis involve ?

- (A) Reducing punctuations as well as grammatical errors to minimalist
- (B) Correct reference citations
- (C) Consistency in the way of thesis writing
- (D) Well defined abstract

Select the answers from the codes given below :

- (a) (B), (C) and (D)
- (b) (A), (B), (C) and (D)
- (c) (A), (B) and (C)
- (d) (A), (B) and (D)

27. On what basis did Jean Piaget give his theory of cognitive development of humans ?

- | | |
|-------------------------|--------------------------|
| (a) Evaluation Research | (b) Fundamental Research |
| (c) Applied Research | (d) Action Research |

28. What are the core elements of a dissertation ?

- (a) Introduction; Data Collection; Data Analysis; Conclusions and Recommendations
- (b) Executive Summary; Literature Review; Data Gathered; Conclusions; Bibliography
- (c) Research Plan; Research Data; Analysis; References
- (d) Introduction; Literature Review; Research Methodology; Results; Discussions and Conclusions

29. "Sampling Cases" can be defined as :
- (a) Sampling using a sampling frame
 - (b) Identifying people who are suitable for research
 - (c) Literally the researcher's brief case
 - (d) A sampling of people, newspapers, television programs etc.
30. Which technique is generally followed when the population is finite ?
- (a) Systematic Sampling Technique
 - (b) Purposive Sampling Technique
 - (c) Area Sampling Technique
 - (d) None of the above
31. Research problem is selected from the standpoint of :
- (a) Social relevance
 - (b) Financial support
 - (c) Researcher's interest
 - (d) Availability of relevant literature
32. Which one among the following is the most comprehensive source of population data ?
- (a) Census
 - (b) National Sample Surveys
 - (c) Demographic Health Surveys
 - (d) National Family Health Surveys

- 33.** The process not needed in experimental research is :
- (a) Controlling
 - (b) Observation
 - (c) Reference collection
 - (d) Manipulation and replication
- 34.** What are those conditions where a research problem is *not* viable ?
- (a) It is new and adds something to knowledge
 - (b) It can be researched
 - (c) It has utility and relevance
 - (d) It contains dependent and independent variables
- 35.** How can we enhance the research objective ?
- (a) By making it more valid
 - (b) By making it more reliable
 - (c) By making it more impartial
 - (d) All of the above
- 36.** Action-research can be understood as :
- (a) A longitudinal research
 - (b) An applied research
 - (c) A kind of research being carried out to solve a specific problem
 - (d) All of the above

37. On what basis can one formulate the assumptions ?
- (a) The cultural background of the country
 - (b) Universities
 - (c) Some specific characteristics of castes
 - (d) All of the above
38. Which one among the following falls under the category of research development ?
- (a) Descriptive Research
 - (b) Philosophical Research
 - (c) Action Research
 - (d) All of these
39. What is the use of Factorial Analysis ?
- (a) For setting the hypotheses
 - (b) To understand the difference between two variables
 - (c) To understand the relationship between two variables
 - (d) To understand the difference between various variables
40. What is the best-suited name for a process that doesn't necessitate experimental research ?
- (a) Manipulation
 - (b) Controlling
 - (c) Content analysis
 - (d) Observation
41. Which one among the following variables cannot be expressed in quantitative terms ?
- (a) Numerical Aptitude
 - (b) Marital Status
 - (c) Socio-economic Status
 - (d) Professional Attitude

42. The "Sociogram" technique is used to study :
- (a) Vocational Interest
 - (b) Human Relations
 - (c) Professional Competence
 - (d) Achievement Motivation
43. Which one among the following phrases does not correspond to the meaning of research as a process ?
- (a) Problem Solving
 - (b) Trial and Error
 - (c) Objective Observation
 - (d) Systematic Activity
44. Research can be classified as :
- (a) Basic, Applied and Action Research
 - (b) Quantitative and Qualitative Research
 - (c) Philosophical, Historical, Survey and Experimental Research
 - (d) All of the above
45. The first step of research is :
- (a) Selecting a problem
 - (b) Searching a problem
 - (c) Finding a problem
 - (d) Identifying a problem
46. A research problem is feasible only when :
- (a) It has utility and relevance
 - (b) It is researchable
 - (c) It is new and adds something to knowledge
 - (d) All of the above

47. Bibliography given in a research report :

- (a) shows vast knowledge of the researcher
- (b) helps those interested in further research
- (c) has no relevance to research
- (d) All of the above

48. Fundamental research reflects the ability to :

- (a) Synthesize new ideals
- (b) Expound new principles
- (c) Evaluate the existing material concerning research
- (d) Study the existing literature regarding various topics

49. The experimental study is based on :

- (a) The manipulation of variables
- (b) Conceptual parameters
- (c) Replication of research
- (d) Survey of literature

50. The main characteristic of scientific research is :

- (a) empirical
- (b) theoretical
- (c) experimental
- (d) All of these

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51. The sequence $\{x_n\}$, where :

$$x_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(n-1)!} \text{ is :}$$

- (a) increasing and bonded
- (b) decreasing and bonded
- (c) increasing and not bonded
- (d) decreasing and not bonded

52. The number of surjective maps from a set of 4 elements to a set of 3 elements is :

- (a) 36
- (b) 64
- (c) 69
- (d) 81

53. Let $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences of real numbers such that the series

$\sum a_n^2$ and $\sum b_n^2$ converges, then the series $\sum a_n b_n$ is :

- (a) Conditionally convergent
- (b) May not convergent
- (c) Absolutely convergent
- (d) Converges to zero but not converges absolutely

54. The Lebesgue measure of the set $\left\{0 < x \leq 1 : x \sin \frac{\pi}{2x} \geq 0\right\}$ is :
- (a) 0 (b) 1
(c) $\ln 2$ (d) $1 - \ln \sqrt{2}$
55. Which of the following real-valued functions on $(0,1)$ is uniformly continuous?
- (a) $f(x) = \frac{1}{x}$ (b) $f(x) = \frac{\sin(x)}{x}$
(c) $f(x) = \sin \frac{1}{x}$ (d) $f(x) = e^{\frac{1}{x}}$
56. A function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ is defined by $f(x, y) = xy$. Let $v = (1, 2)$ and $a = (a_1, a_2)$ be two elements of \mathbb{R}^2 . The directional derivative of f in the direction of v at a is :
- (a) $a_1 + 2a_2$ (b) $a_2 + 2a_1$
(c) $\frac{a_2}{2} + a_1$ (d) $\frac{a_1}{2} + a_2$
57. Let A be a metric space and $A \subseteq X$ be a connected set with at least two distinct points. Then the number of distinct points in A is :
- (a) 2
(b) More than 2, but finite
(c) Countable, infinite
(d) Uncountable

58. Let $\{X, Y, Z\}$ be a basis of \mathbb{R}^3 . Consider the following statements P and Q :

P : $\{X + Y, Y + Z, Z + X\}$ is a basis of \mathbb{R}^3 .

Q : $\{X + Y + Z, X + 2Y - Z, X - 3Z\}$ is a basis of \mathbb{R}^3 .

Which of the above statements hold *true* ?

- (a) Both P, Q
- (b) Only P
- (c) Only Q
- (d) Neither P nor Q

59. Let P be a $n \times n$ matrix with integral entries and $Q = P + \frac{1}{2}I$, where I denotes

the identity matrix. Then, Q is :

- (a) Idempotent
- (b) Invertible
- (c) Nilpotent
- (d) Unipotent

60. Let $A \in \mathbb{C}^{m \times n}$ and A' , A^* denotes transpose and conjugate A respectively.

Then :

- (a) $\text{rank}(AA^* A) = \text{rank}(A)$
- (b) $\text{rank}(A) = \text{rank}(A^2)$
- (c) $\text{rank}(A) = -\text{rank}(A' A)$
- (d) $\text{rank}(A^2) - \text{rank}(A) = \text{rank}(A^3) - \text{rank}(A^2)$

61. A homogeneous system of 5 linear equation in 6 variable admits :
- (a) no solution in \mathbb{R}^6
 - (b) a unique solution in \mathbb{R}^6
 - (c) finite, but more than 2 solutions in \mathbb{R}^6
 - (d) infinitely many solutions in \mathbb{R}^6
62. Let V be the vector space of all 2×2 matrices over \mathbb{R} . Then W consisting of all matrices A for which $A^2 = A$:
- (a) is not a subspace of V as it is not closed
 - (b) is not a subspace of V as it is not closed under vector multiplication, under vector addition
 - (c) Both (a) and (b)
 - (d) is a vector subspace of V
63. Let W_1 and W_2 be finite dimensional space V . If $\dim W_1 = 2$, $\dim W_2 = 2$, $\dim (W_1 + W_2) = 3$, then $\dim (W_1 \cap W_2)$ is :
- (a) 1
 - (b) 2
 - (c) 3
 - (d) 4
64. A is any $n \times n$ matrix with all entries equal to 1, then 0 is an eigen value of A and :
- (a) Multiplicity of 0 is $n - 1$
 - (b) Multiplicity of 0 is 1
 - (c) Multiplicity of 0 is n
 - (d) Multiplicity of 0 is 0

65. The real part of the principal value of 4^{4-i} is :
- (a) $256 \cos(\ln 4)$ (b) $64 \cos(\ln 4)$
(c) $16 \cos(\ln 4)$ (d) $4 \cos(\ln 4)$
66. An analytic function is :
- (a) Finitely differentiable
(b) Not differentiable
(c) Infinitely differentiable
(d) None of the above
67. u, v are called conjugate harmonic functions, if :
- (a) u, v are harmonic functions
(b) u, v are harmonic functions, $u + iv$ may not be analytic function, $u + iv$ is analytic function
(c) u, v are harmonic functions
(d) $u + iv$ is analytic function
68. Consider the power series $\sum_{n=1}^{\infty} z^{n!}$. The radius of convergence of this series is :
- (a) 0
(b) ∞
(c) 1
(d) a real number greater than 1

69. For the function $f(z) = \frac{1 - e^{-z}}{z}$, the point $z = 0$ is :
- (a) an essential singularity
 - (b) a pole of order 0
 - (c) a pole of order one
 - (d) a removable singularity
70. Let f be a bilinear transformation that maps -1 to 1 , i to ∞ , and $-i$ to 0 . Then, $f(1)$ is equal to :
- (a) -2
 - (b) -1
 - (c) i
 - (d) $-i$
71. In a non-abelian group, the element a has order 108. Then the order of a^{42} is :
- (a) 54
 - (b) 27
 - (c) 18
 - (d) 9
72. What is the maximum order of any element in A_{10} ?
- (a) 21
 - (b) 25
 - (c) 24
 - (d) 20
73. The cardinality of the centre of D_{12} is :
- (a) 4
 - (b) 3
 - (c) 1
 - (d) 2

74. The number of 5-Sylow subgroups in the group of order 45 is :
- (a) 1 (b) 2
(c) 3 (d) 4
75. The number of homomorphism from Z_3 to Z_9 is :
- (a) 4 (b) 5
(c) 9 (d) None of these
76. Let R be a ring. If $R[x]$ is a principal ideal domain, then R is necessarily a :
- (a) Unique Factorization Domain
(b) Principal Ideal Domain
(c) Field
(d) Euclidean Domain
77. The remainder when $98!$ is divided by 101 is equal to :
- (a) 22 (b) 50
(c) 25 (d) 55
78. Let F be a field with 7^6 elements and let K be a subfield of F with 49 elements. Then the dimension of F as a vector space over K is :
- (a) 3 (b) 4
(c) 5 (d) 1

79. The order of the permutation (12) (546) (3978) in the symmetric group S_9 is :

- (a) 6
- (b) 12
- (c) 9
- (d) 24

80. The initial value problem $y' = y^{\frac{3}{5}}, y(0) = b$ is :

- (a) a unique solution if $b = 0$
- (b) no solution if $b = 1$
- (c) infinitely many solutions if $b = 2$
- (d) a unique solution if $b = 1$

81. An integrating factor for the differential equation $(2xy + 3x^2 + 6y^3)dx + (x^2 + 6y^2)dy = 0$ is :

- (a) x^3
- (b) y^3
- (c) e^{3x}
- (d) e^{3y}

82. The differential equation $y'' + 2x(y') = 0$, satisfying the condition $y(1) = 0$, $y'(1) = 1$:

- (a) has no solution
- (b) has a unique solution
- (c) has two distinct solutions
- (d) has infinite solutions

83. The singular solution of the differential equation $(xp - y)^2 = p^2 - 1$ is :

(a) $x^2 + y^2 = 1$

(b) $x^2 - y^2 = 1$

(c) $x^2 + 2y^2 = 1$

(d) $x^2 - 2y^2 = 1$

84. The boundary value problem $-u'' = \lambda u$, $0 < x < 1$ and $u'(0) = 0$, $u(1) + u'(1) = 0$ has :

(a) A unique eigen value

(b) Two eigen values

(c) Infinite eigen values

(d) All of the above

85. Let $z = z(x, y)$ be a solution of $\frac{\partial z}{\partial x} \frac{\partial z}{\partial y} = 1$ passing through $(0, 0, 0)$. Then

$z(0, 1)$ is :

(a) 0

(b) 1

(c) 2

(d) 4

86. The partial differential equation $(u_x)^4 u_{xx} + u_{yy} + (u_y)^2 = 0$ is :

(a) Linear and order 4

(b) Quasi-linear and of order 2

(c) Quasi-linear and of order 4

(d) Linear and of order 2

87. Which of the following statement is correct for the partial differential equation :

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$

- (a) It is linear equation of order 2
- (b) It is non-linear equation of order 2
- (c) It is linear equation of order 1
- (d) It is non-linear equation of order 1

88. A complete solution of partial differential equations,

$$x \frac{\partial u}{\partial x} + y \frac{\partial z}{\partial y} - z = \frac{\partial z}{\partial x} \frac{\partial z}{\partial y}$$

- (a) $z = ax + by - ab$; where a and b are arbitrary constants
- (b) $z = x^2 + y^2 - 2ab$; where a and b are arbitrary constants
- (c) $z = ax^2 + by^2 + abxy$; where a and b are arbitrary constants
- (d) $z = ax - by + ab$; where a and b are arbitrary constants

89. Which of the following is elliptic ?

- (a) Heat equation
- (b) Laplace equation
- (c) Wave equation
- (d) None of these

90. A totally disconnected space is a :

- (a) T_0 -space
- (b) T_1 -space
- (c) T_2 -space
- (d) T_3 -space

91. Every indiscrete space is :

- (a) compact and connected (b) not compact
(c) disconnected (d) connected but not compact

92. While solving the equation $x^2 - 3x + 1 = 0$ using the Newton-Raphson Method with the initial guess of a root as 1, the value of the root after one iteration is :

- (a) 1.5 (b) 1
(c) 0.5 (d) 0

93. Using Euler's method taking step size = 0.05, the approximate value of y obtained corresponding to $x = 1.3$ for the initial value problem $\frac{dy}{dx} = \sqrt{y+2x}$, and $y(1.2) = 2$ is :

- (a) 2.21 (b) 3.21
(c) 2.55 (d) 4.2

94. Let $a \in (-1, 1)$ be such that the quadrature rule $\int_{-1}^1 f(x)dx \approx f(-a) + f(a)$ is exact for all polynomials of degree less than or equal to 3. Then $3a^2 =$

- (a) 2 (b) 3
(c) 1 (d) 6

95. Choose the correct :

- (a) $\nabla = 1 - E^{-1}$ (b) $\nabla = 1 + E^{-1}$
(c) $\nabla = -1 + E^{-1}$ (d) $\nabla = -1 - E^{-1}$

96. Order of convergence in Newton-Raphson method is :
- (a) 2 (b) 3
(c) 1 (d) 1.68
97. The resolvent kernel is directly related to the :
- (a) Fourier transform
(b) Laplace transform
(c) Green's function
(d) Sturm-Liouville theory
98. What type of integral equation has a kernel that is a function of both the upper limit and the integration variable ?
- (a) Fredholm-type (b) Volterra-type
(c) Singular-type (d) Differential-type
99. What does the term "momentum" represent in classical mechanics ?
- (a) The force required to stop an object
(b) The resistance of an object to a change in its state of motion
(c) The energy of an object due to its motion
(d) The speed of an object
100. Which law of thermodynamics states that the entropy of an isolated system never decreases over time ?
- (a) Zeroth law (b) First law
(c) Second law (d) Third law